Influences of Space Perturbations on Robotic Assembly Process of Ultra-Large Structures

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Abstract

The space assembly of two flexible beams by a dual-arm space robot is a typical assembly scenario to construct ultra-large space structure. Yet, previous studies mainly focused on the assembly of small structures, neglecting the influences of space perturbations. In this paper, two modelling methods are proposed to study the influences of space perturbations on the space assembly process of ultra-large space structures. Firstly, a theoretical modelling method is proposed based on quasi-static hypothesis and linear structural mechanics. The theoretical model can be utilized for analytically estimating the transverse and axial distributed forces of the flexible beams, structural vibrations, and the control moments of the space robot. An orbit-attitude-structure coupled simulation model is then established to validate the theoretical model and study the dynamic behaviours more accurately, using absolute nodal coordinate formulation and natural coordinate formulation. Finally, the effects of the attitude angle, orbital radius, and lengths of beams on the dynamic responses during assembly are investigated. Theoretical and simulation results show that the control moments and structural vibration amplitude increase dramatically with the length of the beams. The effects of Coriolis force and gravity gradient must be considered for ultra-large space structures during assembly, otherwise the control moments and structural vibrations would be substantially underestimated. The results are instructive to the assembly strategy design as well as modular component design of ultra-large space structures.

1 Introduction

Ultra-large size is one of the most important features for future spacecraft [1]. For instance, the diameter of the spaceport would reach more than 600 m to generate comfortable artificial gravity [2]. And the area of the solar array of the solar power satellites would reach the order of square kilometre to generate electricity for terrestrial or space use [3, 4]. Ultra-large space-based solar reflectors can reflect sunlight to the Earth, Moon, as well as Mars for solar power generation and night-time explorations [5, 6].

Space assembly is the most promising way to construct ultra-large spacecraft. It not only overcomes the mass and volume limitations of the launch vehicle, but also alleviates the effects of launch environmental loads on the whole spacecraft [7]. Compared to self-assembly or assembly by astronaut, the assembly by space robots are more appealing for ultra-large spacecraft. On the one hand, the large structural modules of the ultra-large spacecraft are usually massive and non-autonomous. Space robots are suitable for the extensive and repetitive tasks with high risk and long time [8]. On the other hand, the space robots are more accurate and efficient. Moreover, they can carry out other on-orbit service and maintenance tasks after completing the assembly missions [9].

However, the studies on the dynamics and control of ultra-large space structures during space assembly are not sufficient, which is one of the key technologies for space assembly [10]. Gralla and de Weck compared the advantages of four space assembly strategies for next-generation large space structure, including self-assembly, single tug, multiple tug, and in-space refuelling [11]. They focused on the propellant requirement and orbital design for different assembly strategies. McInnes proposed a distributed orbit and attitude control law for a large space platform assembled from several rigid components [12]. The components were completely controlled by space robots like self-assembly situation. Chen et al. proposed an output consensus control method for a team of self-assembly flexible spacecraft considering collision avoidance requirement [13]. Each flexible spacecraft was modelled by a rigid hub connected with a flexible beam. Literatures [12, 13] focused on the self-assembly control methods for a few components. Yet, the assembly of ultra-large space structure will be a different situation: large modules should be assembled by space robots in sequence. Wang et al. studied a distributed structural vibration control method for a large space structure during assembly [14, 15]. The structure was simplified as a cantilever plate with an increasing number of modules, and the operation disturbance of the space robot was simplified as a 1-N force last for 0.1 s. Cao et al. studied the dynamic modelling and analysis of the assembly process of an orb-shaped solar array [16]. The model was established by the Tschauner-Hempel equation, finite element method, and a classical vehicle-bridge coupled model. However, the operations of the space robots were not involved in the literatures [11–16].

The dynamics and control of space robots were widely studied and many progress were achieved in recent years [17]. However, the studies on the assembly by space robots are limited. Boning and Dubowsky proposed a coordinate control algorithm for a cooperative space robot team to assemble large flexible space structures and conducted ground experiments [18]. Xu et al. studied the dynamic modelling, analysis, and control of a flexible-base space robot capturing a large flexible spacecraft [19]. Meng et al. investigated a vibration suppression method for a space robot with flexible appendages through the control of a manipulator theoretically and experimentally [20, 21]. The experiment demonstrated that the vibrations of flexible appendages were suppressed while the moving target was successfully captured by the space manipulator. Lu et al. studied the ground experiment of the assembly of cooperative modular components using a robot with an economic camera [22]. Ishijima et al. studied the transportation and structural control methods for the transport maneuvering of a large flexible space structure by two space robots [23]. However, the literatures [18–23] mainly concentrated on the assembly of ordinary-size structures without considering space perturbations. The influences of space perturbations increase dramatically with the size of the structure [24]. For instance, the gravity-gradient induced deformation of a Sun-facing beam is proportional to the fifth order of its length [25].

This paper proposes two modelling methods to study the assembly process of two flexible beams by a space robot considering space perturbations. On the one hand, a quasi-static linear model (named theoretical model) is proposed and the analytical results are obtained to make quick estimations of the dynamic characteristics during assembly, including the structural vibrations of the beams and the required control moments of the space robot. On the other hand, a nonlinear and high-dimensional dynamic model (named simulation model) is constructed to study the dynamic behaviours of the assembly process accurately. The flexible beams are modelled using absolute nodal coordinate formulation (ANCF), and the space robot is modelled by natural coordinate formulation (NCF). Gravity gradient and Coriolis force are considered for the two models. Other space perturbations, such as the non-spherical gravity of the Earth, mainly produce uniform acceleration on the system and have little influence on the assembly process. Thus they are not considered in this paper.

The rest of this paper is organized as follows. Section 2 describes the space assembly system and assembly process. Section 3 constructs a theoretical model for the structural distributed forces, structural vibrations, and control moments of the space robot. A simulation model for the assembly system is given in Section 4, including the trajectory planning and control methods for the space robot. Sections 5 and 6 present the comparison of the proposed models and the influences of system parameters. Finally, the main conclusions are summarized in the last section.

2 Problem formulation

The space assembly system consists of a space robot and two flexible beams, as seen in Fig. 1. In order to reduce the complexity of the models and improve simulation efficiency, only the motions in the orbital plane are considered in this paper [26]. The modelling methods can be extended to more general 3-dimensional cases in future works. The flexible beams have been captured tightly by the space robot and stabilized to the ideal initial conditions for the assembly process, i.e., the beams are collinear without relative motion or structural deformations.

A global inertial coordinate system $\mathrm{O}XY$ is constructed, and the origin O is located at the center of the Earth. The joints of the space robot are represented by B – G. Point A and Point H are end effectors. The rigid body DE is regarded as a satellite platform of the space robot. The angles of the joints are denoted by θ_1 - θ_3 and θ_5 - θ_7. The attitude angle of the satellite platform is represented by θ_4, which is approximately equal to the attitude angle of the system α. The flexible beams are denoted by KI and MN. The space robot is considered a multi-rigid-body system because its size and flexibility are small compared to the ultra-large beams. Point A and Point H are coincident with Point M and Point K respectively.

The space robot in this paper is similar to that in Literature [27]. The rigid bodies of the space robot are simplified as cylinders with a diameter of 0.2 m and a density of $1000\ \mathrm{kg/m^3}$. The lengths and masses of the rigid bodies are represented by l_i and m_i, where $i = 1, 2, \cdots, 7$ represents AB, BC, ⋯, and GH respectively. The lengths of rigid bodies are $l_2 = l_3 = l_5 = l_6 = 7\ \mathrm{m}$ and $l_1 = l_4 = l_7 = 2\ \mathrm{m}$. The cross-sectional area, second moment of the cross-section, density, and Young's modulus of the flexible beams are denoted by , , ρ, and respectively. The lengths and masses of Beam MN and Beam KI are represented by L_{MN}, L_{KI}, m_{MN}, and m_{KI}.

The assembly process is shown in Fig. 2. The flexible beams get close along the dashed line MK until they touch each other at point P (not necessarily the midpoint of MK), under the control of the space robot. At the same time, the attitude angle of the space robot θ_4 remains unaltered through attitude control. The gravity, gravity gradient, and inertial forces of the assembly system are considered. The assembly system is initially in a circular orbit with a constant orbital radius r_0 and a constant orbital angular velocity $\omega_0 = \sqrt{\mu / r_0^3}$, where $\mu = 3.986 \times 10^{14}\ \mathrm{m}^3 \cdot \mathrm{s}^{-2}$. The orbital angle is denoted by θ ($\dot{\theta} = \omega_0$, $\ddot{\theta} = 0$). Orbital perturbation or orbital control is not considered.

A local coordinate system Sxy is introduced such that the axis points to Point I, and the center of mass of the assembly system is located at axis. It should be pointed out that Point S is the center of mass of the system on axis and is not fixed to any body, as shown in Fig. 2. The use of Sxy is convenient because Point S can be regarded as an equilibrium point of the gravitational force and inertial force.

3 Theoretical modelling method

A theoretical modelling method is proposed in this section to analyse the dynamic characteristics of the assembly process. Firstly, some assumptions are given to simplify the assembly process. Then the transverse and axial force analyses of the flexible bodies are conducted. Finally, the analytical solutions of the vibrations of the beams and the control moments of the space robot are obtained accordingly.

3.1 Assumptions

The assembly process of ultra-large space structures is a complicated dynamic process involving not only the dynamics of the space robot and large structures, but also the control system of the space robot. In order to make quick estimations on the assembly process, a theoretical model is constructed using quasi-static method and linear structural mechanics based on the following assumptions.

1. Ideal control, including the joints and the attitude, of the space robot is considered. In other words, the control errors of the joints are zero and the flexible beams get close to each other according to a planned trajectory. Based on this assumption, the coupled effects between the space robot and the flexible beams vanish.
2. The size, mass, and moment of inertia of the flexible beams are much larger than the space robot, because this paper concentrates on the assembly of ultra-large structures. Thus, the inertial forces and moments of the space robot can be neglected.
3. The assembly process is very slow because of the large masses of the beams. The assembly time is considerably larger than the largest structural vibration period of the beams. Thus, the dynamic effects of the system can be neglected.
4. Small deformation is assumed to use the linear Euler-Bernoulli beam theory.

3.2 Transverse and axial distributed forces

Based on the above assumptions of ideal control, small robot, slow assembly, and small deformation, the transverse and axial force distributions of the beams during space assembly can be derived using the infinitesimal method. The simplified model is shown in Fig. 3. The transverse and axial forces of Beam KI are firstly derived. The results are then extended to Beam MN.

In order to use the infinitesimal method, an arbitrary point on Beam KI (Point Q) is selected. The coordinate of Point Q is denoted by x_{Q}, in which way the other points are defined. The following notation is introduced to describe the relative coordinate from Point K to Point Q

$$x_{\mathrm{KQ}} = x_{\mathrm{Q}} - x_{\mathrm{K}}$$

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The relative coordinates of other points are denoted in the same way. The relative coordinate from Point S to Point Q x_{SQ} can be calculated by

$$\begin{cases} x_{\mathrm{SQ}} = x_{\mathrm{SP}} + x_{\mathrm{PK}} + x_{\mathrm{KQ}} \\ \dot{x}_{\mathrm{SQ}} = \dot{x}_{\mathrm{PK}}, \ddot{x}_{\mathrm{SQ}} = \ddot{x}_{\mathrm{PK}} \end{cases}$$

2

where x_{SP} is a constant that dependent on the parameters of the assembly system, x_{PK} is time-varying during assembly. The relative coordinate x_{SP} can be calculated through the center of mass of the system after assembly

$$x_{\mathrm{SP}} = \frac{m_{\mathrm{MN}} L_{\mathrm{MN}} - m_{\mathrm{KI}} L_{\mathrm{KI}}}{2\left(m_{\mathrm{MN}} + m_{\mathrm{KI}} + m_{\mathrm{robot}}\right)}$$

3

where m_{robot} is the mass of the space robot. It can be seen that $x_{\mathrm{SP}} = 0$ if Beam MN has the same parameters as Beam KI. In order to calculated x_{PK}, the integral form of the momentum conservation law is used for the assembly process

$$m_{\mathrm{MN}}\left(x_{\mathrm{MK}} - x_{\mathrm{PK}}\right) + m_{\mathrm{robot}}\left(\frac{x_{\mathrm{MK}}}{2} - x_{\mathrm{PK}}\right) = m_{\mathrm{KI}} x_{\mathrm{PK}}$$

4

where x_{MK} is a quintic polynomial of time that will be given in the trajectory planning of the space robot. Thus, x_{PK} is calculated by

$$x_{\mathrm{PK}} = \frac{\left(2m_{\mathrm{MN}} + m_{\mathrm{robot}}\right) x_{\mathrm{MK}}}{2\left(m_{\mathrm{MN}} + m_{\mathrm{KI}} + m_{\mathrm{robot}}\right)}$$

5

The gravitational force vector of an infinitesimal mass element $\mathrm{d}m = \rho A \mathrm{d}x_{\mathrm{KQ}}$ at Point Q is calculated by

$$\mathrm{d}\mathbf{F}_{\mathrm{gravity}} = -\frac{\mu \mathrm{d}m}{r^3}\mathbf{r}$$

6

where $\mathbf{r} = \left[X_{\mathrm{Q}}, Y_{\mathrm{Q}}\right]^{\mathrm{T}}$ is the global position vector of Point Q, and $r = \|\mathbf{r}\|$ is the orbital radius. The inertial force of $\mathrm{d}m$ is

$$\mathrm{d}\mathbf{F}_{\mathrm{inertia}} = -\mathrm{d}m\ddot{\mathbf{r}}$$

7

The global position vector of Point Q and its derivatives are calculated by

$$\mathbf{r} = \begin{bmatrix} X_{\mathrm{Q}} \\ Y_{\mathrm{Q}} \end{bmatrix} = \begin{bmatrix} r_0\cos\theta + x_{\mathrm{SQ}}\cos(\theta + \alpha) \\ r_0\sin\theta + x_{\mathrm{SQ}}\sin(\theta + \alpha) \end{bmatrix}$$

8

$$\dot{\mathbf{r}} = \begin{bmatrix} \dot{X}_{\mathrm{Q}} \\ \dot{Y}_{\mathrm{Q}} \end{bmatrix} = \begin{bmatrix} -r_0\omega_0\sin\theta + \dot{x}_{\mathrm{PK}}\cos(\theta + \alpha) - x_{\mathrm{SQ}}\omega_0\sin(\theta + \alpha) \\ r_0\omega_0\cos\theta + \dot{x}_{\mathrm{PK}}\sin(\theta + \alpha) + x_{\mathrm{SQ}}\omega_0\cos(\theta + \alpha) \end{bmatrix}$$

9

$$\ddot{\mathbf{r}} = \begin{bmatrix} \ddot{X}_{\mathrm{Q}} \\ \ddot{Y}_{\mathrm{Q}} \end{bmatrix} = \begin{bmatrix} -r_0\omega_0^2\cos\theta + \ddot{x}_{\mathrm{PK}}\cos(\theta + \alpha) - 2\dot{x}_{\mathrm{PK}}\omega_0\sin(\theta + \alpha) - x_{\mathrm{SQ}}\omega_0^2\cos(\theta + \alpha) \\ -r_0\omega_0^2\sin\theta + \ddot{x}_{\mathrm{PK}}\sin(\theta + \alpha) + 2\dot{x}_{\mathrm{PK}}\omega_0\cos(\theta + \alpha) - x_{\mathrm{SQ}}\omega_0^2\sin(\theta + \alpha) \end{bmatrix}$$

10

Then the transverse and axial distributed forces of Beam KI are investigated. The transverse and axial forces of $\mathrm{d}m$ are calculated by

$$\mathrm{d}F_{\mathrm{t}} = \left(\mathrm{d}\mathbf{F}_{\mathrm{gravity}} + \mathrm{d}\mathbf{F}_{\mathrm{inertia}}\right) \cdot \mathbf{t}$$

11

$$\mathrm{d}F_{\mathrm{a}} = \left(\mathrm{d}\mathbf{F}_{\mathrm{gravity}} + \mathrm{d}\mathbf{F}_{\mathrm{inertia}}\right) \cdot \mathbf{a}$$

12

where $\mathbf{t} = [-\sin(\theta + \alpha), \cos(\theta + \alpha)]^{\mathrm{T}}$ and $\mathbf{a} = [\cos(\theta + \alpha), \sin(\theta + \alpha)]^{\mathrm{T}}$ are the unit normal vector and unit tangent vector of the undeformed beam respectively, and the positive transverse and axial directions are coincident with axis and axis. Thus, Eqs. (11) and (12) can be rewritten as

$$\frac{\mathrm{d}F_{\mathrm{t}}}{\mathrm{d}m} = -\frac{\mu}{r^3}\left[-X_{\mathrm{Q}}\sin(\theta + \alpha) + Y_{\mathrm{Q}}\cos(\theta + \alpha)\right] + \ddot{X}_{\mathrm{Q}}\sin(\theta + \alpha) - \ddot{Y}_{\mathrm{Q}}\cos(\theta + \alpha)$$

13

$$\frac{\mathrm{d}F_{\mathrm{a}}}{\mathrm{d}m} = -\frac{\mu}{r^3}\left[X_{\mathrm{Q}}\cos(\theta + \alpha) + Y_{\mathrm{Q}}\sin(\theta + \alpha)\right] - \ddot{X}_{\mathrm{Q}}\cos(\theta + \alpha) - \ddot{Y}_{\mathrm{Q}}\sin(\theta + \alpha)$$

14

Substituting Eqs. (8) and (10) into Eqs. (13) and (14) yields

$$\frac{\mathrm{d}F_{\mathrm{t}}}{\mathrm{d}m} = \frac{\mu r_0 \sin\alpha}{r^3} - \omega_0^2 r_0 \sin\alpha - 2\omega_0 \dot{x}_{\mathrm{PK}}$$

15

$$\frac{\mathrm{d}F_{\mathrm{a}}}{\mathrm{d}m} = -\mu\frac{x_{\mathrm{SQ}} + r_0\cos\alpha}{r^3} + r_0\omega_0^2\cos\alpha - \ddot{x}_{\mathrm{PK}} + x_{\mathrm{SQ}}\omega_0^2$$

16

In Eqs. (15) and (16), r_0, ω_0, and α are constant, x_{PK} is time-varying, is determined by the cosine law

$$r^2 = {r_0}^2 + x_{\mathrm{SQ}}^2 + 2x_{\mathrm{SQ}} r_0 \cos\alpha$$

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The first terms of Eqs. (15) and (16) are nonlinear functions of x_{SQ} because of . Actually, the variation of is very small when x_{SQ} varies, because the size of the flexible beam is much smaller than the orbital radius r_0. In order to obtain simple forms of the transverse and axial distributed forces, a first-order Taylor series expansion is adopted to approximate r^{-3} around ${r_0}^{-3}$, which yields

$$\begin{aligned}\frac{\mathrm{d}F_{\mathrm{t}}}{\mathrm{d}m} &= \frac{\mu}{r_0^2}\sin\alpha - \frac{3\mu r_0 \sin\alpha}{2r_0^5} 2x_{\mathrm{SQ}} r_0 \cos\alpha - \omega_0^2 r_0 \sin\alpha - 2\omega_0 \dot{x}_{\mathrm{PK}} \\ &= -\frac{3}{2}\omega_0^2 x_{\mathrm{SQ}} \sin(2\alpha) - 2\omega_0 \dot{x}_{\mathrm{PK}}\end{aligned}$$

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$$\frac{\mathrm{d}F_{\mathrm{a}}}{\mathrm{d}m} = -\left(\frac{\mu}{r_0^3} - \frac{3\mu x_{\mathrm{SQ}}\cos\alpha}{r_0^4}\right)\left(x_{\mathrm{SQ}} + r_0\cos\alpha\right) + r_0\omega_0^2\cos\alpha - \ddot{x}_{\mathrm{PK}} + x_{\mathrm{SQ}}\omega_0^2$$

$$= 3\omega_0^2 x_{\mathrm{SQ}}\cos^2\alpha - \ddot{x}_{\mathrm{PK}}$$

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Thus, the simplified transverse and axial distributed forces of Beam KI are obtained

$$q_{\mathrm{t,KI}} = \frac{\mathrm{d}F_{\mathrm{t}}}{\mathrm{d}x_{\mathrm{KQ}}} = -\frac{3}{2}\rho A\omega_0^2 x_{\mathrm{SQ}}\sin(2\alpha) - 2\rho A\omega_0\dot{x}_{\mathrm{PK}}$$

20

$$q_{\mathrm{a,KI}} = \frac{\mathrm{d}F_{\mathrm{a}}}{\mathrm{d}x_{\mathrm{KQ}}} = 3\rho A\omega_0^2 x_{\mathrm{SQ}}\cos^2\alpha - \rho A\ddot{x}_{\mathrm{PK}}$$

21

The first terms of Eqs. (20) and (21) represent the gravity gradient of Beam KI, which are dependent on the orbital radius and attitude angle. It is linearly distributed along the beam, as shown in Fig. 4. The first term of Eq. (20) is coincident with the result of literature [25]. The second term of Eq. (20) is the Coriolis force of the beam during assembly. It is proportional to the orbital angular velocity and the assembly velocity. It is distributed uniformly along the beam. The second term of Eq. (21) is the acceleration due to the assembly operation.

It can be seen from Eq. (20) that the gravity-gradient term vanishes if $\alpha=0$ or $\alpha=\pi/2$. For the axial distributed force in Eq. (21), the gravity-gradient term reaches a maximum when $\alpha=0$, and becomes 0 when $\alpha=\pi/2$. Thus, $\alpha=\pi/2$ is more desirable in terms of external forces. Considering that $\dot{x}_{\mathrm{PK}} < 0$ for the assembly process, the transverse distributed force could be reduced if $\sin(2\alpha) > 0$, as depicted in Fig. 4.

For Beam MN, the derivation procedure is similar, and the results of the transverse and axial distributed forces are

$$q_{\mathrm{t,MN}} = -\frac{3}{2}\rho A\omega_0^2 x_{\mathrm{QS}}\sin(2\alpha) - 2\rho A\omega_0\dot{x}_{\mathrm{MP}}$$

22

$$q_{\mathrm{a,MN}} = 3\rho A\omega_0^2 x_{\mathrm{QS}}\cos^2\alpha - \rho A\ddot{x}_{\mathrm{MP}}$$

23

where the positive transverse and axial directions of Beam MN are opposite and axes respectively, Point Q is an arbitrary point on Beam MN, and $x_{\mathrm{MP}}=x_{\mathrm{MK}}-x_{\mathrm{PK}}$ is also a quintic polynomial of time.

3.3 Quasi-static vibrations of the beams

Based on the transverse distributed forces of the beams, the quasi-static vibrations can be obtained. By using the Assumption (1) in Section 3.1, the flexible beams can be simplified as cantilever beams, because the transformations and rotations of the captured points of the beams are ideally restricted by the space robot.

Based on Assumption (4), the quasi-static vibration of Beam KI is governed by

$$EI\frac{\mathrm{d}^4 v_{\mathrm{KI}}}{\mathrm{d}x_{\mathrm{KQ}}^4} = q_{\mathrm{t}}\left(x_{\mathrm{KQ}}\right)$$

24

The boundary conditions of the cantilever beam can be written as

$$v_{\mathrm{KI}}(0) = \frac{\mathrm{d}v_{\mathrm{KI}}}{\mathrm{d}x_{\mathrm{KQ}}}(0) = \frac{\mathrm{d}^2 v_{\mathrm{KI}}}{\mathrm{d}x_{\mathrm{KQ}}^2}(L_{\mathrm{KI}}) = \frac{\mathrm{d}^3 v_{\mathrm{KI}}}{\mathrm{d}x_{\mathrm{KQ}}^3}(L_{\mathrm{KI}}) = 0$$

25

Then, the quasi-static vibration of the beam can be obtained by integrating Eq. (24)

$$v_{\mathrm{KI}} = -\frac{3}{2}\frac{\rho A}{EI}\omega_0^2\left(\frac{1}{120}x_{\mathrm{KQ}}^5 - \frac{L_{\mathrm{KI}}^2}{12}x_{\mathrm{KQ}}^3 + \frac{L_{\mathrm{KI}}^3}{6}x_{\mathrm{KQ}}^2\right)\sin(2\alpha)$$

$$-\frac{3}{2}\frac{\rho A}{EI}\omega_0^2\left(\frac{1}{24}x_{\mathrm{KQ}}^4 - \frac{L_{\mathrm{KI}}}{6}x_{\mathrm{KQ}}^3 + \frac{L_{\mathrm{KI}}^2}{4}x_{\mathrm{KQ}}^2\right)\left(x_{\mathrm{SP}} + x_{\mathrm{PK}}\right)\sin(2\alpha)$$

$$-2\frac{\rho A}{EI}\omega_0\dot{x}_{\mathrm{PK}}\left(\frac{1}{24}x_{\mathrm{KQ}}^4 - \frac{L_{\mathrm{KI}}}{6}x_{\mathrm{KQ}}^3 + \frac{L_{\mathrm{KI}}^2}{4}x_{\mathrm{KQ}}^2\right)$$

26

Thus, the vibration of Point I is

$$v_{\mathrm{I}} = -\frac{11}{80}\frac{\rho A L_{\mathrm{KI}}^{5}}{EI}\omega_0^2 \sin(2\alpha) - \frac{3}{16}\frac{\rho A L_{\mathrm{KI}}^{4}}{EI}\omega_0^2\left(x_{\mathrm{SP}} + x_{\mathrm{PK}}\right)\sin(2\alpha) - \frac{1}{4}\frac{\rho A L_{\mathrm{KI}}^{4}}{EI}\omega_0 \dot{x}_{\mathrm{PK}}$$

27

According to the third term of Eq. (27), it should be emphasized that the deformation of the beam is nonzero even if the attitude angle remains 0, which is a unique characteristic of the space assembly process. The maximum deformation is at least proportional to L_{KI}^{4} if x_{PK} does not change with L_{KI}. Therefore, the vibrations of the ultra-large beam during space assembly must be considered seriously. Similarly, the vibration of Beam MN is

$$v_{\mathrm{MN}} = -\frac{3}{2}\frac{\rho A}{EI}\omega_0^2\left(\frac{1}{120}x_{\mathrm{QM}}^5 - \frac{L_{\mathrm{MN}}^2}{12}x_{\mathrm{QM}}^3 + \frac{L_{\mathrm{MN}}^3}{6}x_{\mathrm{QM}}^2\right)\sin(2\alpha)$$

$$-\frac{3}{2}\frac{\rho A}{EI}\omega_0^2\left(\frac{1}{24}x_{\mathrm{QM}}^4 - \frac{L_{\mathrm{MN}}}{6}x_{\mathrm{QM}}^3 + \frac{L_{\mathrm{MN}}^2}{4}x_{\mathrm{QM}}^2\right)\left(x_{\mathrm{PS}} + x_{\mathrm{MP}}\right)\sin(2\alpha)$$

$$-2\frac{\rho A}{EI}\omega_0 \dot{x}_{\mathrm{MP}}\left(\frac{1}{24}x_{\mathrm{QM}}^4 - \frac{L_{\mathrm{MN}}}{6}x_{\mathrm{QM}}^3 + \frac{L_{\mathrm{MN}}^2}{4}x_{\mathrm{QM}}^2\right)$$

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And the vibration of Point N is

$$v_{\mathrm{N}} = -\frac{11}{80}\frac{\rho A L_{\mathrm{MN}}^{5}}{EI}\omega_0^2 \sin(2\alpha) - \frac{3}{16}\frac{\rho A L_{\mathrm{MN}}^{4}}{EI}\omega_0^2\left(x_{\mathrm{PS}} + x_{\mathrm{MP}}\right)\sin(2\alpha) - \frac{1}{4}\frac{\rho A L_{\mathrm{MN}}^{4}}{EI}\omega_0 \dot{x}_{\mathrm{MP}}$$

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3.4 Control moments of the space robot

Based on the transverse and axial distributed forces of the beams, the resultant forces and moments of the beams on the space robot are studied. The positive directions of the resultant forces and moments are shown in Fig. 5. They are calculated by

$$M_{\mathrm{KI}} = \int_0^{L_{\mathrm{KI}}} x_{\mathrm{KQ}} q_{\mathrm{t,KI}}\left(x_{\mathrm{KQ}}\right)\mathrm{d}x_{\mathrm{KQ}}$$

$$= -L_{\mathrm{KI}}^2 \rho A \omega_0\left[\dot{x}_{\mathrm{PK}} + \frac{1}{2}L_{\mathrm{KI}}\omega_0 \sin(2\alpha) + \frac{3}{4}\omega_0\left(x_{\mathrm{SP}} + x_{\mathrm{PK}}\right)\sin(2\alpha)\right]$$

30

$$F_{x,\mathrm{KI}} = \int_0^{L_{\mathrm{KI}}} q_{\mathrm{a},\mathrm{KI}}\left(x_{\mathrm{KQ}}\right)\mathrm{d}x_{\mathrm{KQ}}$$

$$= \frac{\rho A L_{\mathrm{KI}}}{2}\left[3L_{\mathrm{KI}}{\omega_0}^2\cos^2(\alpha) + 6{\omega_0}^2\left(x_{\mathrm{SP}} + x_{\mathrm{PK}}\right)\cos^2(\alpha) - 2\ddot{x}_{\mathrm{PK}}\right]$$

31

$$F_{y,\mathrm{KI}} = \int_0^{L_{\mathrm{KI}}} q_{\mathrm{t},\mathrm{KI}}\left(x_{\mathrm{KQ}}\right)\mathrm{d}x_{\mathrm{KQ}}$$

$$= -\frac{\rho A L_{\mathrm{KI}}\omega_0}{4}\left[8\dot{x}_{\mathrm{PK}} + 3L_{\mathrm{KI}}\omega_0\sin(2\alpha) + 6\omega_0\left(x_{\mathrm{SP}} + x_{\mathrm{PK}}\right)\sin(2\alpha)\right]$$

32

$$M_{\mathrm{MN}} = \int_0^{L_{\mathrm{MN}}} x_{\mathrm{QM}} q_{\mathrm{t,MN}}\left(x_{\mathrm{QM}}\right)\mathrm{d}x_{\mathrm{QM}}$$

$$= -L_{\mathrm{MN}}^2\rho A\omega_0\left[\dot{x}_{\mathrm{MP}} + \frac{1}{2}L_{\mathrm{MN}}\omega_0\sin(2\alpha) + \frac{3}{4}\omega_0\left(x_{\mathrm{MP}} + x_{\mathrm{PS}}\right)\sin(2\alpha)\right]$$

33

$$F_{x,\mathrm{MN}} = \int_0^{L_{\mathrm{MN}}} q_{\mathrm{a,MN}}\left(x_{\mathrm{QM}}\right)\mathrm{d}x_{\mathrm{QM}}$$

$$= \frac{\rho A L_{\mathrm{MN}}}{2}\left[3L_{\mathrm{MN}}{\omega_0}^2\cos^2(\alpha) + 6{\omega_0}^2\left(x_{\mathrm{MP}} + x_{\mathrm{PS}}\right)\cos^2(\alpha) - 2\ddot{x}_{\mathrm{MP}}\right]$$

34

$$F_{y,\mathrm{MN}} = \int_0^{L_{\mathrm{MN}}} q_{\mathrm{t,MN}}\left(x_{\mathrm{QM}}\right)\mathrm{d}x_{\mathrm{QM}}$$

$$= -\frac{\rho A L_{\mathrm{MN}}\omega_0}{4}\left[8\dot{x}_{\mathrm{MP}} + 3L_{\mathrm{MN}}\omega_0\sin(2\alpha) + 6\omega_0\left(x_{\mathrm{MP}} + x_{\mathrm{PS}}\right)\sin(2\alpha)\right]$$

35

The joint control moments of the space robot can be estimated roughly. Based on the Assumption (2), the inertial forces and moments of the arms are not considered to simplify the control moment estimation. The control moments M_1 - M_7 are defined corresponding to the joint angles θ_1 - θ_7. The positive directions of M_1 - M_7 are selected such that they increase the joint angles θ_1 - θ_7. According to the force analysis diagram in Fig. 5, the required control moments M_5 - M_7 can be calculated by

$$M_7 = -M_{\mathrm{KI}} - F_{x,\mathrm{KI}} l_7$$

36

$$M_6 = M_7 + F_{x,\mathrm{KI}} l_6 \cos\left(\theta_7\right) - F_{y,\mathrm{KI}} l_6 \sin\left(\theta_7\right)$$

37

$$M_5 = M_6 - F_{x,\mathrm{KI}} l_5 \cos\left(\theta_6 + \theta_7\right) + F_{y,\mathrm{KI}} l_5 \sin\left(\theta_6 + \theta_7\right)$$

38

The control moments M_1 - M_3 are calculated by

$$M_1 = M_{\mathrm{MN}} - F_{x,\mathrm{MN}} l_1$$

39

$$M_2 = M_1 + F_{x,\mathrm{MN}} l_2 \cos\left(\theta_1\right) + F_{y,\mathrm{MN}} l_2 \sin\left(\theta_1\right)$$

40

$$M_3 = M_2 - F_{x,\mathrm{MN}} l_3 \cos\left(\theta_1 + \theta_2\right) - F_{y,\mathrm{MN}} l_3 \sin\left(\theta_1 + \theta_2\right)$$

41

The attitude control moment of the satellite platform of the space robot can be calculated by

$$M_4 = -M_{\mathrm{KI}} - M_{\mathrm{MN}} - F_{y,\mathrm{KI}} x_{\mathrm{MK}}$$

42

The above control moments are useful in the controller design of the space robot.

4 Simulation modelling method

To study the space assembly process more accurately, a simulation modelling method is proposed based on ANCF and NCF. The assumptions in Section 3.1 can be abandoned in the simulation model. Gravity gradient and Coriolis force are taken into account by adopting the second-order Taylor series expansion of the gravitational potential energy of ANCF/NCF and simulating the orbital and attitude motions. Thus, the proposed simulation modelling method is an orbit-attitude-structure coupled modelling method.

4.1 Two dimensional ANCF beam element

The flexible beams are modeled by ANCF, which is a well-known rigid-flexible coupled modelling method that is able to deal with large-deformation and large-rotation cases [28–30] such as soft robot[31], flexible membrane system [32] and fluid materials [33–35]. The two-dimensional two-node Euler-Bernoulli ANCF beam element is adopted [36]. The basic theory of this ANCF beam element is briefly introduced in this section.

The global position vector of an arbitrary point in an element can be calculated by the following cubic interpolation

$$\mathbf{r}\left(x_{\mathrm{e}}\right) = [X, Y]^{\mathrm{T}} = \mathbf{S}\left(x_{\mathrm{e}}\right)\mathbf{e}$$

43

where $x_{\mathrm{e}} \in \left[0, l_{\mathrm{e}}\right]$ is the local coordinate of the centerline of the element, l_{e} is the length of the element, $\mathbf{S}\left(x_{\mathrm{e}}\right)$ is a cubic shape function, and $\mathbf{e}$ is the generalized coordinate vector. The shape function can be found in literature [36]. The generalized coordinate vector is defined as

$$\mathbf{e} = \left[\mathbf{r}^{\mathrm{T}}(0), \frac{\partial \mathbf{r}^{\mathrm{T}}(0)}{\partial x_{\mathrm{e}}}, \mathbf{r}^{\mathrm{T}}\left(l_{\mathrm{e}}\right), \frac{\partial \mathbf{r}^{\mathrm{T}}\left(l_{\mathrm{e}}\right)}{\partial x_{\mathrm{e}}}\right]^{\mathrm{T}}$$

44

The dynamic equations of the element can be derived through Hamilton’s equations. To this end, the kinetic energy and potential energy are introduced. According to Eq. (43), the velocity vector of a point in an element is

$$\dot{\mathbf{r}}\left(x_{\mathrm{e}}\right) = \mathbf{S}\left(x_{\mathrm{e}}\right)\dot{\mathbf{e}}$$

45

Therefore, the kinetic energy of an element can be expressed as

$$T_{\mathrm{e}} = \frac{1}{2}\int_0^{l_{\mathrm{e}}} \rho A \dot{\mathbf{r}}^{\mathrm{T}}\dot{\mathbf{r}}\mathrm{d}x_{\mathrm{e}} = \frac{1}{2}\dot{\mathbf{e}}^{\mathrm{T}}\int_0^{l_{\mathrm{e}}} \rho A \mathbf{S}^{\mathrm{T}}\left(x_{\mathrm{e}}\right)\mathbf{S}\left(x_{\mathrm{e}}\right)\mathrm{d}x_{\mathrm{e}}\dot{\mathbf{e}} = \frac{1}{2}\dot{\mathbf{e}}^{\mathrm{T}}\mathbf{M}_{\mathrm{e}}\dot{\mathbf{e}}$$

46

where $\mathbf{M}_{\mathrm{e}}$ is the mass matrix of the element. The elastic potential energy of an element can be calculated by

$$U_{\mathrm{ela,e}} = \frac{1}{2}\int_0^{l_\mathrm{e}} EA\left(\frac{\partial u}{\partial x_\mathrm{e}}\right)^2 + EI\left(\frac{\partial^2 v}{\partial x_\mathrm{e}^2}\right)^2 \mathrm{d}x_\mathrm{e}$$

47

where and are the axial deformation and transverse deformation respectively. By defining the axial direction, Eq. (47) can be expressed as a nonlinear function of the generalized coordinates. The detailed expression of the elastic energy is not given for simplicity, which is found in literature [36].

The key modelling procedure is to calculate the distributed gravitational force and gravity gradient on the flexible bodies. The generalized gravitational force of an element can be calculated by

$$\begin{cases} \mathbf{f}_{\mathrm{gra,e}} = -\dfrac{\partial U_{\mathrm{gra,e}}}{\partial \mathbf{e}} \\ U_{\mathrm{gra,e}} = -\displaystyle\int_0^{l_\mathrm{e}} \frac{\mu\rho A}{\left(\mathbf{r}^\mathrm{T}\mathbf{r}\right)^{1/2}} \mathrm{d}x_\mathrm{e} \end{cases}$$

48

where $U_{\mathrm{gra,e}}$ is the gravitational potential energy of the element. However, it is not easy to obtain the theoretical expression of $\mathbf{f}_{\mathrm{gra,e}}$ because of the nonlinearity of the integrant. The approximate expression of $\mathbf{f}_{\mathrm{gra,e}}$ can be obtained by using the second-order Taylor series expansion of $U_{\mathrm{gra,e}}$, which finally yields

$$\mathbf{f}_{\mathrm{gra,e}} = \frac{\mu}{\left(\mathbf{r}_{0,\mathrm{e}}^\mathrm{T}\mathbf{r}_{0,\mathrm{e}}\right)^{5/2}}\left(\mathbf{A}_\mathrm{e}\mathbf{e} + \mathbf{B}_\mathrm{e}^\mathrm{T}\right)$$

49

where $\mathbf{r}_{0,\mathrm{e}}$ is the global position vector of the center of mass of the element. The matrixes $\mathbf{A}_\mathrm{e}$ and $\mathbf{B}_\mathrm{e}$ are calculated by

$$\mathbf{A}_{\mathrm{e}} = \frac{\rho A l_{\mathrm{e}}}{420}\begin{bmatrix} 156\mathbf{D}_{\mathrm{e}} & 22l_{\mathrm{e}}\mathbf{D}_{\mathrm{e}} & 54\mathbf{D}_{\mathrm{e}} & -13l_{\mathrm{e}}\mathbf{D}_{\mathrm{e}} \\ & 4l_{\mathrm{e}}^2\mathbf{D}_{\mathrm{e}} & 13l_{\mathrm{e}}\mathbf{D}_{\mathrm{e}} & -3l_{\mathrm{e}}^2\mathbf{D}_{\mathrm{e}} \\ & & 156\mathbf{D}_{\mathrm{e}} & -22l_{\mathrm{e}}\mathbf{D}_{\mathrm{e}} \\ \text{symmetric} & & & 4l_{\mathrm{e}}^2\mathbf{D}_{\mathrm{e}} \end{bmatrix}$$

50

$$\mathbf{B}_{\mathrm{e}} = \rho A l_{\mathrm{e}}\begin{bmatrix} \frac{1}{2}\mathbf{r}_{0,\mathrm{e}}^{\mathrm{T}} & \frac{l_{\mathrm{e}}}{12}\mathbf{r}_{0,\mathrm{e}}^{\mathrm{T}} & \frac{1}{2}\mathbf{r}_{0,\mathrm{e}}^{\mathrm{T}} & -\frac{l_{\mathrm{e}}}{12}\mathbf{r}_{0,\mathrm{e}}^{\mathrm{T}} \end{bmatrix}$$

51

where $\mathbf{D}_{\mathrm{e}} = 3\mathbf{r}_{0,\mathrm{e}}\mathbf{r}_{0,\mathrm{e}}^{\mathrm{T}} - \mathbf{r}_{0,\mathrm{e}}^{\mathrm{T}}\mathbf{r}_{0,\mathrm{e}}\mathbf{I}_2$, and $\mathbf{I}_2$ is a 2-dimensional identity matrix.

4.2 Two dimensional NCF

The multi-rigid-body system of the space robot is modeled by NCF. NCF is a straightforward modelling method that all rigid bodies are described in a global inertial coordinate system [37, 38]. The NCF were widely used in dynamic studies of spacecraft and robotics [39–41]. NCF were applied to three-dimensional cases in the previous studies. In this section, a two-dimensional NCF is presented to model the space robot.

In the two-dimensional NCF, the global coordinates of two points and one vector are used to describe a rigid body. The modelling procedure of Rigid body AB is taken as an example, as shown in Fig. 6. The global position vector of an arbitrary point in AB can be expressed as

$$\mathbf{r} = [X, Y]^{\mathrm{T}} = \mathbf{C}\left(x_1, y_1\right)\mathbf{e}_1$$

52

$$\mathbf{e}_1 = \left[\mathbf{r}_{\mathrm{A}}, \mathbf{r}_{\mathrm{B}}, \mathbf{v}_1\right]^{\mathrm{T}}$$

53

where x_1 and y_1 are local coordinates of the rigid body, $\mathbf{C}\left(x_1, y_1\right)$ is a linear shape function, $\mathbf{e}_1$ is the generalized coordinate vector, and $\mathbf{v}_1$ is a unit normal vector fixed to Rigid body AB and perpendicular to AB. The expression of the shape function is

$$\mathbf{C}\left(x_1, y_1\right) = \left[\frac{x_1}{l_1}\mathbf{I}_2, \left(1 - \frac{x_1}{l_1}\right)\mathbf{I}_2, y_1\mathbf{I}_2\right]$$

54

There are 3 degrees of freedom for a planar rigid body. However, there are 6 generalized coordinates in Eq. (53). Thus, they are subjected to the following 3 constraints that describe the length and perpendicularity of the vectors

$$\begin{cases} \left(\mathbf{r}_{\mathrm{A}} - \mathbf{r}_{\mathrm{B}}\right)^{\mathrm{T}}\left(\mathbf{r}_{\mathrm{A}} - \mathbf{r}_{\mathrm{B}}\right) - l_1^2 = 0, \\ \mathbf{v}_1^{\mathrm{T}}\mathbf{v}_1 - 1 = 0, \\ \left(\mathbf{r}_{\mathrm{A}} - \mathbf{r}_{\mathrm{B}}\right)^{\mathrm{T}}\mathbf{v}_1 = 0 \end{cases}$$

55

Similar to the ANCF, the kinetic energy of Rigid body AB is calculated by

$$T_1 = \frac{1}{2}\int_{V_1} \rho\dot{\mathbf{r}}^{\mathrm{T}}\dot{\mathbf{r}}\mathrm{d}V_1 = \frac{1}{2}\dot{\mathbf{e}}_1^{\mathrm{T}}\left[\int_{V_1} \rho\mathbf{C}^{\mathrm{T}}\left(x_1, y_1\right)\mathbf{C}\left(x_1, y_1\right)\mathrm{d}V_1\right]\dot{\mathbf{e}}_1 = \frac{1}{2}\dot{\mathbf{e}}_1^{\mathrm{T}}\mathbf{M}_1\dot{\mathbf{e}}_1$$

56

where V_1 and $\mathbf{M}_1$ are the volume and the mass matrix of Rigid body AB. The generalized gravitational force of the NCF can be calculated by the same procedure as the ANCF element using the second-order Taylor series expansion of the gravitational potential energy. The final expression of the generalized gravitational force of NCF is

$$\mathbf{f}_{\mathrm{gra},1} = -\frac{\partial U_{\mathrm{gra},1}}{\partial \mathbf{e}} = \frac{\mu}{\left(\mathbf{r}_{0,1}^{\mathrm{T}}\mathbf{r}_{0,1}\right)^{5/2}}\left(\mathbf{A}_1\mathbf{e}_1 + \mathbf{B}_1^{\mathrm{T}}\right)$$

57

where $\mathbf{r}_{0,\mathrm{e}}$ is the global position vector of the center of mass of Rigid body AB, $U_{\mathrm{gra},1}$ is the gravitational potential energy. The matrix $\mathbf{A}_1$ and $\mathbf{B}_1$ are calculated by

$$\mathbf{A}_1 = \frac{1}{l_1^2}\begin{bmatrix} \left(m_1 l_1^2 + I_{x1} - 2m_1 l_1 x_{\mathrm{G1}}\right)\mathbf{D}_1 & \left(m_1 l_1 x_{\mathrm{G1}} - I_{x1}\right)\mathbf{D}_1 & \left(m_1 y_{\mathrm{G1}} l_1^2 - I_{xy1} l_1\right)\mathbf{D}_1 \\ & I_{x1}\mathbf{D}_1 & I_{xy1} l_1 \mathbf{D}_1 \\ \text{symmetric} & & I_{y1} l_1^2 \mathbf{D}_1 \end{bmatrix}$$

58

$$\bar{\mathbf{B}} = m_1 \begin{bmatrix} \frac{1}{2}\mathbf{r}_{0,1}^{\mathrm{T}} & \frac{l_1}{12}\mathbf{r}_{0,1}^{\mathrm{T}} & \frac{1}{2}\mathbf{r}_{0,1}^{\mathrm{T}} & -\frac{l_1}{12}\mathbf{r}_{0,1}^{\mathrm{T}} \end{bmatrix}$$

59

where $I_{x1} = \int_{V_1} \rho x_1^2 \mathrm{d}V_1$, $I_{y1} = \int_{V_1} \rho y_1^2 \mathrm{d}V_1$, and $I_{xy1} = \int_{V_1} \rho x_1 y_1 \mathrm{d}V_1$ can be calculated from the inertia matrix with respect to $\mathrm{A}x_1 y_1$ coordinate system, $x_{\mathrm{G1}} = \int_{V_1} \rho x_1 \mathrm{d}V_1$ and $y_{\mathrm{G1}} = \int_{V_1} \rho y_1 \mathrm{d}V_1$ are the center of mass in $\mathrm{A}x_1 y_1$, and $\mathbf{D}_1 = 3\mathbf{r}_{0,1}\mathbf{r}_{0,1}^{\mathrm{T}} - \mathbf{r}_{0,1}^{\mathrm{T}}\mathbf{r}_{0,1}\mathbf{I}_2$.

4.3 Dynamic modelling of the space assembly system

Based on the above ANCF and NCF, the orbit-attitude-structure coupled dynamic equations of the space assembly system are derived in this subsection, based on the constrained Hamilton's equations.

The space robot consists of 7 rigid bodies with 9 degrees of freedom. According to Section 4.2, the space robot should contain 42 generalized coordinates and 33 constraints, including the internal constraints of each rigid body and the linear constraints between the adjacent rigid bodies. However, the global position vector of the common point of two adjacent rigid bodies appears only once in the generalized coordinate vector. Therefore, the generalized coordinate vector of the space robot is

$$\mathbf{q}_{\mathrm{robot}} = \left[\mathbf{r}_{\mathrm{A}}^{\mathrm{T}}, \mathbf{r}_{\mathrm{B}}^{\mathrm{T}}, \mathbf{v}_1^{\mathrm{T}}, \mathbf{r}_{\mathrm{C}}^{\mathrm{T}}, \mathbf{v}_2^{\mathrm{T}}, \mathbf{r}_{\mathrm{D}}^{\mathrm{T}}, \mathbf{v}_3^{\mathrm{T}}, \mathbf{r}_{\mathrm{E}}^{\mathrm{T}}, \mathbf{v}_4^{\mathrm{T}}, \mathbf{r}_{\mathrm{F}}^{\mathrm{T}}, \mathbf{v}_5^{\mathrm{T}}, \mathbf{r}_{\mathrm{G}}^{\mathrm{T}}, \mathbf{v}_6^{\mathrm{T}}, \mathbf{r}_{\mathrm{H}}^{\mathrm{T}}, \mathbf{v}_7^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathrm{R}^{30}$$

60

The constraints of the space robot are as follows

$$\begin{cases} \left(\mathbf{r}_{i1} - \mathbf{r}_{i2}\right)^{\mathrm{T}}\left(\mathbf{r}_{i1} - \mathbf{r}_{i2}\right) = l_i^2 \\ \mathbf{v}_i^{\mathrm{T}}\mathbf{v}_i = 1 \\ \left(\mathbf{r}_{i1} - \mathbf{r}_{i2}\right)^{\mathrm{T}}\mathbf{v}_i = 0 \end{cases} \quad i = 1, 2, \ldots, 7$$

61

where $\mathbf{r}_{i1}$ and $\mathbf{r}_{i2}$ represent the global position vectors of the left and right ends of each rigid body, for instance, $\mathbf{r}_{11}$ and $\mathbf{r}_{12}$ represent $\mathbf{r}_{\mathrm{A}}$ and $\mathbf{r}_{\mathrm{B}}$ for $i = 1$. The constraints in Eq. (61) can also be abbreviated as

$$\mathbf{g}\left(\mathbf{q}_{\mathrm{robot}}\right) = \mathbf{0} \in \mathrm{R}^{21}$$

62

For the whole space assembly system, the generalized coordinate vector is

$$\mathbf{q} = \left[\mathbf{q}_{\mathrm{robot}}^{\mathrm{T}}, \mathbf{q}_{\mathrm{MN}}^{\mathrm{T}}, \mathbf{q}_{\mathrm{KI}}^{\mathrm{T}}\right]^{\mathrm{T}}$$

63

where $\mathbf{q}_{\mathrm{MN}}$ and $\mathbf{q}_{\mathrm{KI}}$ are the generalized coordinates of Beam MN and Beam KI respectively. The kinetic energy of the system can be expressed as

$$T = \sum_{i=1}^{7} T_i + T_{\mathrm{MN}} + T_{\mathrm{KI}} = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{q}}$$

64

The mass matrix of the system $\mathbf{M}$ in Eq. (64) can be superposed by the mass matrices of the ANCF elements and the rigid bodies. Similarly, the potential energy of the system is

$$U = \sum_{i=1}^{7} U_{\mathrm{gra},i} + U_{\mathrm{gra,MN}} + U_{\mathrm{gra,KI}} + U_{\mathrm{ela,MN}} + U_{\mathrm{ela,KI}}$$

65

The grasping relationships between the space robot and the flexible beams are equivalent to spring-damper systems for simplicity. The grasping force and moment of Point A of Rigid body AB are calculated by

$$\mathbf{F}_{\mathrm{A}} = k\left(\mathbf{r}_{\mathrm{M}} - \mathbf{r}_{\mathrm{A}}\right) + c\left(\dot{\mathbf{r}}_{\mathrm{M}} - \dot{\mathbf{r}}_{\mathrm{A}}\right)$$

66

$$M_{\mathrm{A}} = k\left(\alpha_{\mathrm{AM}} - \frac{\boldsymbol{\pi}}{2}\right) + c\dot{\alpha}_{\mathrm{AM}}$$

67

where $\mathbf{r}_{\mathrm{M}}$is the first two generalized coordinates of $\mathbf{q}_{\mathrm{MN}}$, α_{AM} is the angle between AB and MN. The coefficients $k = 10^6$ and $c = 10^4$ are selected for displacements, while $k = 2 \times 10^7$ and $c = 2 \times 10^4$ are used for rotations. They are selected by experience such that the grasping displacements and rotations are small enough to be ignored, because this paper does not focus on the grasping dynamics. The angle α_{AM} can be calculated by the following expression because it is approximately $\pi/2$

$$\alpha = \arccos\left(\mathbf{v}_{\mathrm{MN}}^{\mathrm{T}} \frac{\mathbf{r}_{\mathrm{B}} - \mathbf{r}_{\mathrm{A}}}{l_1}\right)$$

68

where $\mathbf{v}_{\mathrm{NM}}$ represents the unit tangent vector of the beam MN at point M obtained by the 3rd and 4th generalized coordinates of $\mathbf{q}_{\mathrm{MN}}$. The expressions of grasping forces and moments of GH, MN, and KI can be obtained similarly.

Based on the kinetic energy and potential energy, the dynamic equations of the space assembly system are derived by using the constrained Hamiltonian principle, which yields

$$\begin{cases} \dot{\mathbf{q}} = \mathbf{M}^{-1}\mathbf{p}, \\ \dot{\mathbf{p}} = -\dfrac{\partial U(\mathbf{q})}{\partial \mathbf{q}} - \dfrac{\partial \mathbf{g}^{\mathrm{T}}(\mathbf{q})}{\partial \mathbf{q}}\lambda + \mathbf{Q}, \\ \mathbf{g}(\mathbf{q}) = \mathbf{0} \end{cases}$$

69

where $\mathbf{p}$ is termed the generalized momentum vector of the system, λ is a vector of Lagrangian multiplier, $\mathbf{Q}$ is a generalized external force vector obtained by the principle virtual work. The gravitational force and gravity gradient of the space assembly system are considered in Eq. (69) by using the second-order Taylor series expansion of gravitational potential energy. The initial orbital and attitude conditions r_0, $\dot{r}_0 = 0$, $\theta_0 = 0$, $\dot{\theta}_0 = \sqrt{\mu / r_0^3}$, α_0, and $\dot{\alpha}_0 = 0$ are used to simulate the orbit and attitude of the system. The inertial forces of the system, including the transverse distributed Coriolis forces and axial distributed inertia forces of the beams, are automatically taken into account. Thus, Eq. (69) is an orbit-attitude-structure coupled dynamic model. The energy- and constraint-conserving algorithm in literature [42] is used to solve the differential-algebraic equation set (69).

4.4 Trajectory planning and control

Trajectory planning and trajectory tracking control of the space robot are not the focus of this paper. However, they should be considered in order to accomplish the assembly process. Under the control of the space robot, the flexible beams should get close to each other smoothly. Besides, the motions of the flexible beams should be strictly along axis to avoid exciting the vibrations of the beams to the maximum extent. This requirement can be satisfied by the ingenious cooperative trajectory planning and control of the joints such that the coordinate of the center of mass of the space robot is unaltered during assembly.

The trajectories are firstly planned in the $\mathrm{S}xy$ coordinate system, and then transferred into the joint space through nonlinear geometric relationships. The quintic polynomial is used to plan the relative coordinate from Point M to Point K

$$x_{\mathrm{MK}}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

70

with the following conditions

$$\begin{cases} x_{\mathrm{MK}}(0) = x_{\mathrm{MK0}}, \dot{x}_{\mathrm{MK}}(0) = 0, \ddot{x}_{\mathrm{MK}}(0) = 0, \\ x_{\mathrm{MK}}\left(t_{\mathrm{end}}\right) = 0, \dot{x}_{\mathrm{MK}}\left(t_{\mathrm{end}}\right) = 0, \ddot{x}_{\mathrm{MK}}\left(t_{\mathrm{end}}\right) = 0 \end{cases}$$

71

where t_{end} is the assembly duration, $x_{\mathrm{MK0}} = 18\ \mathrm{m}$ is the initial assembly distance used in this paper, and a_i can be determined by the initial and terminal conditions. Then, the space robot keeps a symmetrical configuration during the assembly process. Arms AB and GH are perpendicular to the flexible beams. Thus, the following relationships can be obtained

$$\begin{cases} \theta_7=\theta_1,\ \theta_6=\theta_2,\ \theta_5=\theta_3, \\ \theta_1+\theta_2+\theta_3=\dfrac{5\pi}{2} \end{cases}$$

72

The two unknowns in Eq. (72) are θ_1 and θ_2, which can be obtained through geometric relationships. In addition, $\theta_4 = \alpha$ is the given attitude angle in the simulations. The relative coordinate from Point M to Point K can also be calculated by the joint angles

$$x_{\mathrm{MK}} = l_4 + 2l_2\sin\theta_1 - 2l_3\sin\left(\theta_1+\theta_2\right)$$

73

In order to maintain the coordinates of Point M and Point K $y_{\mathrm{M}} = y_{\mathrm{K}} = 0$, the center of mass of the space robot, should be unchanged in the simulation, which is calculated by

$$y_{\mathrm{cm,robot}} = l_1 + \frac{-\left(m_2 l_2 + 2m_3 l_2 + m_4 l_2\right)\cos\theta_1 + \left(m_3 l_3 + m_4 l_3\right)\cos\left(\theta_1 + \theta_2\right)}{2m_2 + 2m_3 + m_4}$$

74

In this paper, $y_{\mathrm{cm,robot}} = 6\ \mathrm{m}$ is adopted. It should be pointed out that the trajectory planning methods of space robots with a floating base, such as the virtual arm method [43], are not used because relative trajectories are required for the assembly process, instead of absolute trajectories in [43]. By solving nonlinear Eqs. (73) and (74), θ_1 and θ_2 can be obtained. The other joint angles can be calculated through Eq. (72). The planned trajectories of the joint angles are shown in Fig. 7.

The simple feedforward-feedback control method is used to track the planned trajectories of the joints of the space robot. The control moments are calculated by

$$M_i = K_{\mathrm{FF}} M_{\mathrm{theory},i} + K_{\mathrm{p}i} e_i + K_{\mathrm{d}i} \dot{e}_i$$

75

where $i = 1, 2, \cdots, 7$ ($i = 4$ is the attitude control of the space robot, and other values are joint control), $M_{\mathrm{theory},i}$ denote the control moments of the theoretical model given in Eqs. (36)−(42), K_{FF} is a switch parameter for the feedforward control moment, e_i are the control errors of θ_i, $K_{\mathrm{p}i}$ and $K_{\mathrm{d}i}$ are the proportional and derivative gains respectively. The parameter $K_{\mathrm{FF}} = 1$ means that the controller is a feedforward-feedback controller, while $K_{\mathrm{FF}} = 0$ represents a pure feedback controller. The errors are defined as

$$e_i = \theta_{i,\,\mathrm{planned}} - \theta_i$$

76

where $\theta_{i,\,\mathrm{planned}}$ represent the planned results, and θ_i can be calculated by the generalized coordinates in numerical simulations. The following proportional and derivative gains are adopted for simplicity

$$\begin{cases} K_{\mathrm{p}i} = 1.0 \times 10^6, i = 1, 2, \cdots, 7 \\ K_{\mathrm{d}i} = 2.0 \times 10^4, i = 1, 2, \cdots, 7 \end{cases}$$

77

5 Numerical validation and simulations

This section compares the numerical results of the theoretical model and the simulation model to show the validation of the two modelling methods. The flexible beams KI and MN are composed of n_{KI} and

n_{MN} structural modules. The parameters of a structural module are shown in Table 1. The lengths of the flexible beams are calculated by

$$L_{\mathrm{KI}} = n_{\mathrm{KI}} L_{\mathrm{module}},\ L_{\mathrm{MN}} = n_{\mathrm{MN}} L_{\mathrm{module}}$$

78

The parameters of nine typical cases are given in Table 2. Case 1 is a typical case that other cases are based on Case 1. Case 2 adopts a pure feedback controller. Case 3 studies the effects of the attitude angle α. Case 4 and Case 5 investigate the influences of the numbers of structural modules for the flexible beams. Case 6 – Case 9 further study the effects of the Coriolis force and gravity gradient. The LEO in Table 2 represents low Earth orbit that $r_0 = 7136636\ \mathrm{m}$ and $\omega_0 = (2\pi/6000)\ \mathrm{rad/s}$. The time step in simulations is 0.001 s, the assembly duration is $t_{\mathrm{end}} = 300\ \mathrm{s}$, and the simulation time is 500 s. The space robot implements the assembly task in the first 300 s, and remains stationary in the following 200 s. The simulation results of Case 1 are shown in Fig. 8 - Fig. 16.

Table 1
Parameters of a structural module

Parameter	Value
L_{module}	100 m
	$0.03\ \mathrm{m}^2$
	$4.15 \times 10^{-4}\ \mathrm{m}^4$
ρ	$1000\ \mathrm{kg \cdot m^{-3}}$
	230 GPa

Table 2
Nine cases in the simulations

Case	n_{MN}	n_{KI}	K_{FF}	α	Orbit
Case 1	1	1	1	0 deg	LEO
Case 2	1	1	0	0 deg	LEO
Case 3	1	1	1	80 deg	LEO
Case 4	1	5	1	0 deg	LEO
Case 5	5	5	1	0 deg	LEO
Case 6	1	Vary	1	0 deg	$r_0 = \infty$
Case 7	n_{KI}	Vary	1	0 deg	$r_0 = \infty$
Case 8	1	5	1	Vary	LEO
Case 9	1	5	1	Vary	GEO

The grasping errors of displacements and rotations for Case 1 are shown in Fig. 8. It can be seen that the grasping errors remain very small in the simulation. Thus, the influences of the grasping errors on the simulation results are negligible.

The control errors of the space robot in Case 1 and Case 2 are depicted in Fig. 9 and Fig. 10 respectively. It can be seen that the control errors for Case 1 are smaller than 10^{-4} deg, which reveals the high accuracy of the feedforward-feedback controller. The control errors for Case 2 are about 20 times larger than those of Case 1. In addition, the errors of the stationary stage (the last 200 s) in the simulation are comparatively smaller than the assembly stage (the first 300 s) for both cases. The control moments of the theoretical model and the simulation model in Case 1 and Case 2 are given in Fig. 11 and Fig. 12. The maximum control moments are a little less than 40 N·m. It is worth noting that the control moments of the stationary stage are nonzero because of the axial gravity-gradient distributed forces of the beams. The control moments of the theoretical model for Case 1 are consistent with the simulation model. This phenomenon indicates the validation of the proposed two models. Besides, there is an obvious fluctuation in the control moments of the simulation model for Case 2. It can be concluded that the feedforward control moments improve the control accuracies greatly. Thus, the feedforward-feedback controller is adopted in the following simulations.

The control results in the $\mathrm{S}xy$ coordinate system are presented in Fig. 13 and Fig. 14. It can be found that the control results in the simulations can track the planned results precisely. Particularly, the displacement in axis is very small such that the flexible beams move in the direction strictly. It means that the operation of the space robot is well planned and controlled to avoid the transverse displacements as well as structural vibrations of the flexible beams. This technique is useful for space robots assembling ultra-large space structures. Besides, the motions of flexible beams in direction satisfy the quintic polynomial to accomplish smooth assembly. The control errors in both axis and axis can also be reduced greatly by using the feedforward control moments.

The structural vibrations of Beam KI and Beam MN are illustrated in Fig. 15 and Fig. 16. The maximum deformations are less than 1 mm for the two cases. The vibrations of Beam KI are similar to Beam MN because they have the same parameters. In these two cases, the theoretical structural vibrations are induced by the Coriolis force during assembly, because the attitude angle $\alpha = 0$ and the transverse gravity-gradient forces vanish, as seen in Eqs. (27) and (29).

The structural vibrations for Case 1 match the theoretical model, while the vibrations of Case 2 are superposed by a small-amplitude vibration. The difference is due to the controller of the space robot. As can be seen in Fig. 11 and Fig. 12, the theoretical control moments at $t = 0$ are nonzero due to the axial distributed gravity-gradient force, as shown in Eqs. (31), (34), and (36)–(41). The nonzero control moments at the beginning can be provided by the feedforward control moments for Case 1, which leads to the high control accuracies. On the contrary, these control moments need to be provided by the control errors for the feedback controller in Case 2.

It should be mentioned that the period of the additional vibrations for Case 2 (about 22 s) is larger than the free vibration period of the flexible beams with cantilever boundary (about 10 s), because the vibrations of the beams are coupled with the control of the space robot. However, the assembly duration is much larger the vibration period in simulation, and Assumption (3) is satisfied.

The above simulation results show the validity of the proposed modelling methods because the results of the two models are consistent with each other. By using the control moments of the theoretical model as feedforward control moments, the control accuracies are improved by 20 times, the structural vibration amplitudes are reduced, while the control moments become smoother.

6 Dynamic characteristics for different parameters

The dynamic characteristics of the system during assembly process are analyzed for different parameters in this section, including the effects of the attitude angle, orbital radius, and lengths of the flexible beams.

6.1 Effects of the attitude angle

In order to study the effects of the attitude angle, the assembly process is performed with an attitude angle of $\alpha = 80\ \mathrm{deg}$ (Case 3 of Table 2). The simulation results are depicted in Fig. 17 - Fig. 19. It can be seen that the control of the space robot is very accurate in Case 3 although the control errors are slightly larger than Case 1. In terms of control moments, it is interesting that the trends and the variation amplitudes of the control moments in Case 3 are similar to those in Case 1, except that the maximum absolute value of the control moments in Case 3 are considerably smaller than Case 1. The same phenomenon can also be clearly seen in the structural vibrations in Case 3 in Fig. 19. By using the attitude angle $\alpha = 80\ \mathrm{deg}$, the vibration curve of Case 1 in Fig. 15 is reduced as a whole.

This phenomenon can be explained by the transverse distributed force of Beam KI (Eq. (20) and Fig. 4). The structural vibrations of Case 1 are mainly induced by the distributed Coriolis force because $\alpha = 0$. In Case 3, the transverse gravity-gradient force is in the opposite direction of the Coriolis forces by using a suitable attitude angle, i.e., the external forces of the flexible beams are counteracted each other. Besides, the distributed Coriolis force depends on time, while the transverse gravity-gradient force depends on attitude angle. That is the reason for the similarities between Case 1 and Case 3 in terms of control moments and structural vibrations.

In addition, it should also be pointed out that the differences between the theoretical model and the simulation model in Case 3 are larger than Case 1. The reason is that the initial deformation of the beam in the theoretical model is nonzero due to the transverse distributed force for nonzero attitude angle. However, the beams of the simulation model are undeformed according to Section 2. In other words, the differences between the two models in Case 3 can be attributed to the different initial conditions of the two models. The differences would increase greatly with the lengths of the beams, as will be shown in the following simulations. This problem can be addressed by further considering the effects of the initial conditions in the theoretical model or by giving appropriate initial deformations of the beams in the simulation model.

Based on the parameters of Case 3, the dynamic characteristics during space assembly are studied by changing the attitude angle from $-90\ \mathrm{deg}$ to $90\ \mathrm{deg}$. The theoretical and simulation results are shown in Fig. 20 - Fig. 22. The vibrations of Point N coincide with Point I, which are not shown for simplicity.

The maximum control moments depend largely on the attitude angle because of the influences of the gravity gradient. The control moments reach the minimum values when the attitude angle is around $\alpha = 10\ \mathrm{deg}$ or $\alpha = 80\ \mathrm{deg}$, because the gravity gradient and the Coriolis force counteract each other to the maximum extent. The maximum control moments appear at $\alpha = -45\ \mathrm{deg}$ when the gravity gradient reaches a maximum value with the same direction of the Coriolis force.

The control moments at the end of the assembly process reflect the influence of the gravity gradient on the system, which are shown in Fig. 21 for different attitude angle. It can be seen that the control moments at the end of the assembly process become 0 when $\alpha = \pm 90\ \mathrm{deg}$, which is an unstable equilibrium point of the gravity gradient. For the stable equilibrium attitude angle $\alpha = 0\ \mathrm{deg}$, the axial distributed gravity-gradient force is nonzero and thus the control moments are also nonzero.

In terms of the maximum structural vibrations of the flexible beams, the variation trends are similar to the maximum control moments, with minimum values at $\alpha = 13\ \mathrm{deg}$ or $\alpha = 77\ \mathrm{deg}$. In addition, most of the simulation results agree well with the theoretical results in Fig. 20 - Fig. 22. Whereas, there are noticeable errors in the maximum structural vibrations from $\alpha = 20\ \mathrm{deg}$ to $\alpha = 80\ \mathrm{deg}$ in Fig. 22. Actually, the maximum absolute value of v_{I} from $\alpha = 20\ \mathrm{deg}$ to $\alpha = 80\ \mathrm{deg}$ is the minimum value of

v_{I}. Otherwise, it is the maximum value of v_{I}. The reason for the noticeable errors in Fig. 22 from $\alpha = 20$ deg to $\alpha = 80$ deg is the differences of initial conditions of the two models, as explained in Fig. 19.

6.2 Effects of structural parameters

In this subsection, the effects of structural parameters on the dynamic characteristics of the assembly system are analyzed. Particularly, the flexibility of a beam is sensitive to its length. Besides, the lengths of the beams would change when the beams are composed of different numbers of structural modules. Thus, the dynamic responses are studied for different numbers of structural modules n_{KI} and n_{MN}.

The dynamic responses of Case 4 when $n_{\mathrm{MN}}=1$ and $n_{\mathrm{KI}}=5$ are shown in Fig. 23 - Fig. 25. It can be seen that the maximum values of M_4 - M_7 are increased to about 5 times of the maximum control moments in Fig. 11, while the growths of M_1 - M_3 are unobvious. Besides, in the last 200 s of the simulation, the control moments M_4 - M_7 are not settled under the influence of structural vibrations of Point I, as seen in Fig. 24. The structural vibration amplitude of Beam KI is much larger than that of Beam MN.

The results of the two models are coincident in terms of the control moments M_1 - M_3 and the structural vibrations of Point N, while the errors between the two models are obvious for M_4 - M_7 and v_{I}, especially for the last 200 s.

The dynamic response of Case 5 when $n_{\mathrm{MN}}=n_{\mathrm{KI}}=5$ are illustrated in Fig. 26 and Fig. 27. In this case, the errors of all control moments between the two models are apparent. All control moments are increased greatly compared to Fig. 11. The control moments M_4 - M_7 are much larger than those of Case 4, although they have the same lengths of Beam KI. The structural vibrations of Point I are also increased to about 3 times of Case 4. The reason is that the center of mass of the system x_{SP} and the assembly distance of Beam KI x_{PK} are altered greatly when n_{MN} increase from 1 to 5, according to Eq. (3) and Eq. (5). The vibrations of Point N are consistent with Point I.

Based on Case 4 and Case 5, the dynamic behaviors during assembly are studied for different value of n_{KI}, which are depicted in Fig. 28 - Fig. 32. It can be seen from Fig. 28 - Fig. 30 that the maximum control moments increase linearly with n_{KI}. The maximum absolute value of v_{I} increases rapidly with n_{KI}, and the increasing rate becomes larger for large n_{KI}. On the other hand, the growth of the maximum absolute value of v_{N} is much slower, and it tends to converge to a small value.

The dynamic characteristics are also studied when both n_{KI} and n_{MN} increase, as shown in Fig. 31 and Fig. 32. The growths of the control moments and the structural vibrations are much more severe than Fig. 28 and Fig. 29. The control moments in Fig. 31 are approximately 8 times of the moments in Fig. 28. And the vibrations in Fig. 32 are about 4 times larger than Fig. 29. The results indicate that the control of the space robot would become more and more difficult during the whole assembly process of an ultra-large beam, because the control moments and structural vibrations increase rapidly. However, the problems of large control moments and structural vibrations can be released considerably if the structural modules are assembled one by one. The simulation results are meaningful to the assembly strategy design as well as modular component design of ultra-large space structures.

It can also be seen in Fig. 28 - Fig. 32 that the errors between the theoretical model and the simulation model increase remarkably when $n_{\mathrm{KI}} > 5$. The reason is that the flexibilities of the flexible beams increase with the length. When n_{KI}=5, the vibration period of Beam KI (assumed cantilever boundary conditions) becomes 250 s, which approaches the assembly duration (300 s) in the simulation. Thus, Assumption (3) in Section 3.1 is no longer satisfied. The distributed forces of the beams cannot be considered as quasi-static. Thus, a more complicated dynamic model should be constructed for the fast assembly operations. Increasing the assembly duration can reduce the control moments, structural vibrations, as well as the errors between the two models.

6.3 Effects of the Coriolis force

This subsection aims to compare the maximum control moments between the assembly considering space perturbations and the assembly without space perturbations, because space perturbations were not considered in many previous studies of the assembly dynamics and control [13, 44]. The assembly without space perturbation can be obtained by setting $r_0 = \infty$ and $\omega_0 = 0$. Theoretical and simulation results of the control moments in Case 6 and Case 7 are shown in Fig. 33 and Fig. 34. Structural vibrations are not induced ideally in Case 6 and Case 7, and the structural vibration amplitudes of the simulation model are less than 3 mm.

It is seen that the maximum control moments of Case 6 increase very slowly compared to those in Fig. 28. The control moments of Fig. 28 are about 6 times larger than Fig. 33. The results in Fig. 28 are mainly influenced by the Coriolis force, because the attitude angle $\alpha = 0\ \mathrm{deg}$. However, the effects of Coriolis force or gravity gradient are not considered in Fig. 33. Similarly, the maximum control moments of Case 7 increase linearly with n_{KI}, which are much lower than those in Fig. 31. Thus, it can be concluded that the control moments of the space robot would be underestimated greatly if the Coriolis force is not considered during the assembly process, which would lead to failures of the assembly mission.

6.4 Effects of the orbital radius

Finally, the effects of orbital radius are studied by comparing the results of Case 8 and Case 9. The orbital conditions of GEO are $r_0 = 42164142\ \mathrm{m}$ and $\omega_0 = 7.292124 \times 10^{-5}$. The theoretical and simulation results are presented in Fig. 35 - Fig. 38. It can be seen that the maximum control moments are about 1200 $\mathrm{N} \cdot \mathrm{m}$ in LEO, while they become less than 50 $\mathrm{N} \cdot \mathrm{m}$ in GEO, i.e. the maximum value is reduced by more than 24 times. Moreover, the control moments in GEO are less influenced by the attitude angle (and the gravity gradient). In high-Earth orbit, the effects of Coriolis force should also be considered because the effects of the Coriolis force is only decrease with ω_0, while the effects of the gravity gradient decrease with ω_0^2. Similarly, the structural vibration amplitudes of Point I in GEO are reduced by about 100 times, and are less influenced by the attitude angle. The errors of the control moments and structural vibrations between the two models are mainly due to the difference of initial structural deformations as discussed in Section 5. It can be concluded that the assembly of ultra-large beams in high orbits is more desirable because of low gravity gradient and Coriolis force.

7 Conclusions

This paper studies the dynamics and control of the assembly process of two large flexible beams by a space robot considering the gravity gradient and Coriolis force. A theoretical modelling method and a simulation modelling method are proposed. The effects of space perturbations and system parameters on the control moments of the space robot and the structural vibrations of the beams are studied. The following conclusions can be drawn according to theoretical and simulation results.

1. The dynamic responses of the proposed theoretical model and simulation model agree well to each other under the premise of ideal control, small robot, slow assembly, and small deformation. The theoretical control moments can be used as feedforward control moments for the space robot to greatly improve control accuracy.
2. When the numbers of structural modules of both flexible beams increase synchronously, the control moments and structural vibrations increase dramatically. And the increasing rate becomes faster and faster as the numbers rise. When the number of structural modules of one beam increases while the other beam contains only one structural module, the above problem can be relieved significantly. Thus, the structural modules should be assembled one by one to reduce control moments and structural vibrations.
3. If the effects of Coriolis force and gravity gradient were not considered, the control moments and structural vibrations during space assembly would be underestimated substantially.
4. By adjusting the attitude angle in the assembly process, the effects of the Coriolis force and gravity gradient can be countered each other to some extent. A positive pitch attitude angle is more preferable than a negative value, because the effects of the Coriolis force and the gravity gradient would be superposed for a negative value.
5. The gravity gradient is proportional to ω_0^2 while the Coriolis force is proportional to ω_0. Thus, both the control moments and structural vibrations can be reduced greatly in GEO compared to LEO.

Declarations

Declarations

Conflict of Interest

The authors declare that they have no conflict of interest. This article does not contain any studies with human participants or animals performed by any of the authors. Informed consent was obtained from all individual participants included in the study.

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